



Wood End Park Academy

'To question is to grow'

Calculation Policy



September 2015

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Our Calculation Policy

This calculation policy has been created to meet the expectations of the new national curriculum but most importantly the learning needs of our children at WEPA. The methods chosen match the national curriculum but have also been specifically selected after consideration of our children's learning styles.

Age Expectations

The policy has been organised by year group, considering the national curriculum 2014 expectations. The new curriculum focuses on skills and is not about moving children on to the next method as soon as they can do the one before. Working on more complex and richer problems rather than new methods will support this 'mastering' of maths. However, some children will be working at levels well above their age and will require the introduction of new methods.

Mental Methods

The written methods in this document are important but they by no means replace the mental methods we have developed. As children become more mature and confident with their calculation, they need to think about the most appropriate method to answer the question: mental method, written jottings or formal written method.

Breadth, Challenge and Application

Although initially these methods should be taught thoroughly without context focussing on children's deep understanding of the method; children should also have plenty of opportunity in applying their understanding to other problems within numeracy and across the curriculum.

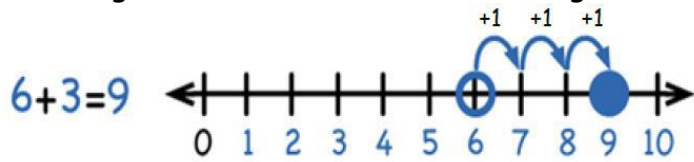
When first introduced to word problems children should use the KOVEMAC steps in order to help them select the correct method and operation(s).

Key words **O**peration **V**isualise **E**stimate **M**ethod **A**nswer **C**heck

Addition

Year 1: Adding numbers to 20

Children should use numbered number lines to add, by counting on in ones. Encourage children to start with the larger number and count on.



As well as using a number line children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different con-texts.
- Read and write the addition (+) and equals (=) signs within number sentences.
- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: $8 + 3 = \square$
 $15 + 4 = \square$ $5 + 3 + 1 = \square$ $\square + \square = 6$
- Use bead strings or bead bars to visualise bridging through 10s e.g. $8 + 5 =$ can be solved by counting on 2 then counting on 3:



Key vocabulary

Add, more, plus, and, make, altogether, total, equal to, equals, double, most count on, number line

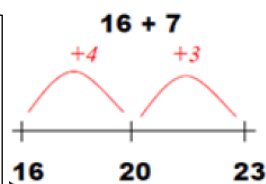
Key Skills

- Reading and writing numbers to 100 in numerals.
- Writing numbers to 20 in words including correct spelling.
- Counting to and across 100 in ones.
- Counting in multiples of 2, 5 and 10.
- Solving simple one step addition problems: using objects, numberlines and images to support.

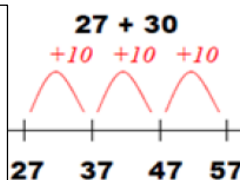
Year 2: Adding two digit numbers

In year 2 children should develop mental fluency with addition and place value involving two digit numbers, then establish more formal methods.

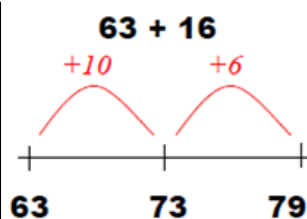
1. Adding two digit numbers and units



2. Adding two digit numbers and tens



3. Adding pairs of two digit numbers, moving to the partitioned column method when secure adding tens and units.



2	0	+	3	
+	3	0	+	4
<hr/>				
5	0	+	7	
<hr/>				
			= 57	

Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

4. Once children are confident they can start using the partitioning method to add numbers that bridge the tens and hundreds boundaries.

5	0	+	8
4	0	+	3
<hr/>			
9	0	+	11
<hr/>			
			= 101

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key Skills

- Add a 2 digit number and units and a 2 digit number and 10s.
- Add pairs of 2 digit numbers.
- Add three single digit number.
- Know and show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and multiple of 10 bonds to 100.
- Count in steps of 2,3 and 5 and count in 10s from any number.
- Understand the place value of 2 -digit numbers (tens and ones).
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve contextual addition problems.

Year 3: Add numbers with up to three digits

In year 3 we will move to the traditional column method and to support this, children will first apply their partitioning skills to the partitioning column method.

1. Introduce the partitioning column method with numbers that do not bridge so children become confident with the method itself.

$$246 + 132 = 378$$

$$\begin{array}{l} 200 + 40 + 6 \\ \underline{100 + 30 + 2} \\ 300 + 70 + 8 = 378 \end{array}$$

2. Once confident, children can start using the partitioning column method to solve problems that bridge the tens and hundreds

$$337 + 188 = 525$$

$$\begin{array}{l} 300 + 30 + 7 \\ \underline{100 + 80 + 8} \\ 400 + 110 + 15 = 525 \end{array}$$

3. Now children are ready to move on to the traditional column methods. Introduce this initially with numbers that do not bridge any boundaries. It is important children remember that it is three hundred add one hundred, NOT 3 + 1

$$116 + 343 = 459$$

$$\begin{array}{r} 343 \\ + 116 \\ \hline 459 \end{array}$$

4. Once the method is secure children are now ready to be introduced to 'carrying' which happens when bridging in the column method. Make sure children add the units first and 'carry' numbers under the bottom line.

$$245 + 84 = 329$$

$$\begin{array}{r} 245 \\ + 84 \\ \hline 329 \\ 1 \end{array}$$

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact

Key skills

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, and ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Year 4: Add numbers with up to four digits

In year 4 children will consolidate their use of the traditional column method and will be able to use it confidently to add numbers up to 4 digits. This could include carrying units, tens and hundreds. In year 4 children should use and apply using contexts including money and measures.

$$\begin{array}{r} 3517 \\ + 396 \\ \hline 3913 \\ \text{,} \quad \text{,} \end{array}$$

Children should already be familiar with the column method from year 3 but it is very important to go over the method again ensuring children understand why they start with the units, have to carry a number etc

Remember:

- Start with the units
- Carry numbers underneath the bottom line
- Place value of each digit e.g. the five is 500 not 5!

Key vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, **thousands, hundreds, digits, inverse**

Key skills

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

Year 5: Adding with more than 4 digits

In year 5 children will now use the column method to add decimal numbers in the context of money and measures. It is important that children have place value skills beyond 4 digits here and fully understand what a decimal number represents.

$$\begin{array}{r} \text{€ } 23.59 \\ + \text{€ } 7.55 \\ \hline \text{€ } 31.14 \end{array}$$

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

Numbers should exceed 4 digits.

$$\begin{array}{r} 23481 \\ + 1362 \\ \hline 24843 \end{array}$$

$$\begin{array}{r} 19.01 \\ 3.65 \\ + 0.70 \\ \hline 23.36 \end{array}$$

Pupils should be able to add more than two values, carefully aligning place value columns.

Empty decimal places can be filled with zero to show the place value in each column.

Key vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry“, expanded, compact, vertical, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths

Key skills:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

Year 6: Add several numbers of increasing complexity

In year 6 children need to use all the previous adding skills developed to add several numbers with a variety of different decimal places. Many of these problems will be in the context of money or measures

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ \small 2 \quad 1 \quad 2 \end{array}$$

Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

Zeros could be added into any empty decimal places, to show there is no value to add.

Children should add several numbers with more than four digits

$$\begin{array}{r} 81,059 \\ 3,668 \\ 15,301 \\ + 20,551 \\ \hline 120,579 \\ \small 1 \quad 1 \quad 1 \quad 1 \end{array}$$

Key vocabulary

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, carry, expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy. □ Pupils understand how to add mentally with larger numbers and calculations of increasing complexity

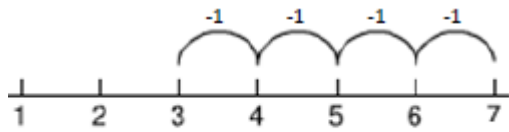
Subtraction

Year 1: Subtracting from numbers to 20

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Subtracting by taking away

Consolidate understanding of subtraction practically before moving on to using number tracks then numbered number lines and hundred squares to subtract by counting back in ones and tens.



$$7 - 4 = 3$$

Beadstrings or beadbars can be used to illustrate subtraction including bridging through 10 by counting back 3 and then counting back another 2.

$$13 - 5 = 8$$

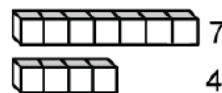


Find the 'difference between'

This should be introduced practically first with an emphasis on the language 'find the difference between' and 'how many more'



Tom has 5 bears. Sara has 3 bears.
How many more bears does Tom have?



What is the difference between seven and four?

Number Families

By the end of Year 1 children should be able to
Recall and use facts within and to 20



If we know $4 + 5 = 9$ We also know:

$$5 + 4 = 9$$

$$9 - 5 = 4$$

$$9 - 4 = 5$$

$$14 + 5 = 19$$

$$19 - 14 = 5, \text{ etc}$$

Subtract using patterns of known facts

E.g. $7 - 3 = 4$ so we know $17 - 3 = 14$,

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?

Key Skills

- Given a number, say one more or one less.
- Count to and over 100, forward and back from any number in 1s.
- Represent and use subtraction facts to 20 and within 20.
- Subtract with one digit and 2 digit numbers to 20, including zero.
- Solve one step problems that involve subtraction using objects, pictures and numbered lines.
- Read and write numbers to 100 in numerals.
- Write numbers in words to 20s, including correct spelling.

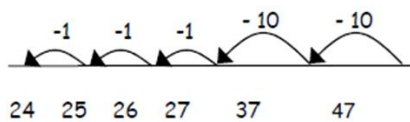
Year 2: Subtracting with two digit numbers

In year 2 children will start to use blank numberlines to subtract by counting back which will greatly support the development of mental subtraction skills. Children should start by physically making and carrying out the calculation using base 10, Numicon, hundred squares or other apparatus then compare their practical version with the written form to develop conceptual understanding.

Using then number line children should:

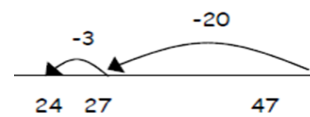
- 2-digit numbers subtract units (by taking away / counting back) e.g. $36-7$
- 2-digit numbers subtract tens (by taking away / counting back) e.g. $48-30$
- Subtracting pairs of 2-digit numbers (see below:)

Subtracting Pairs of two digit numbers:



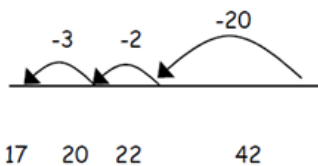
$$47 - 23.$$

Partition the second number into tens and ones. Subtracting the tens first



Move towards more efficient jumps.

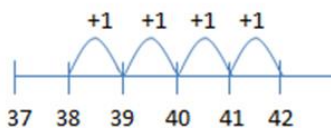
Combine methods with use of a hundred square to reinforce understanding of number value and order.



Teaching children to bridge through ten can help them to become more efficient, for example $42-25$:

Mental Strategies - counting on

$$42 - 38 = 4$$



Children are taught to recognise that when numbers are close together, it is more efficient to count on the difference. They need to be clear about the relationship between addition and subtraction

Key vocabulary

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units

Key skills

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.

Year 3: Subtracting with 2 and 3-digit numbers.

In year three children should be introduced to the Partitioned column method

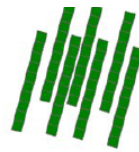
Step 1: Once children are confident in subtracting mentally and using a numberline they should be introduced to the partitioned column method:

$$89 - 35 = 54$$

$$\begin{array}{r} 80 + 9 \\ - 30 + 5 \\ \hline 50 + 4 \end{array}$$

To begin with use examples where no exchanging is required.

Step 2: Introduce exchanging through practical subtraction. Make the larger number with Base 10/Dienes, then subtract 47 from it.



$$\begin{array}{r} 70 + 2 \\ - 40 + 7 \\ \hline 20 + 5 = \underline{25} \end{array}$$

When learning to exchange, explore partitioning in different ways so that pupils understand that when you exchange, the VALUE is the same ie $72 = 70 + 2 = 60 + 12 = 50 + 22$ etc. Emphasise that the value hasn't changed, we have just partitioned it in a different way.

Step 3: Once pupils are secure with the understanding of "exchanging", they can use the partitioned column method to subtract any 2 and 3-digit numbers.

2	3	8	-	1	4	6	=	9	2
2	0	0	+	1	3	0	+	8	
-	1	0	0	+	4	0	+	6	
				0	+	9	0	+	2

Key vocabulary:

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit

Key skills

- Subtract mentally a: 3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract,

Year 4: Subtracting with numbers up to 4 digits

Children will consolidate their knowledge of the partitioning column method for subtraction with 4 digit numbers including those where exchanging is required. Once they are secure with this they will move on to the compact (traditional) method of column subtraction and apply this to a variety of problems including measures and money.

Step 1: reinforce this method and extend by moving towards more complex numbers. Use base 10, place value counters and coins to reinforce concepts and understanding. Ensure method is applied to money and measures using KOVEMAC.

2	7	5	4	-	1	5	6	2	=	1	1	9	2						
2	0	0	0	+	7 00	+	50	+	4	-	£	4	+	30p	+	6p			
-	1	0	0	0	+	5	0	0	+	60	+	2	-	£	2	+	50p	+	3p
	1	0	0	0	+	1	0	0	+	90	+	2	-	£	1	+	80p	+	3p

Step 2: once children are secure with 'exchanging' up to 4-digits, they can move on to the compact column method to subtract up to 4-digit numbers. Begin by asking children to complete a subtraction calculation using the partitioned col-umn subtraction and then display the compact version. Dis-cuss what is the same, what is the different and the benefits of each method. Ensure method is applied to money and measures using KOVEMAC.

2	7	5	4
-	1	5	6
	1	1	9
			2

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse.

Key Skills

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select a mental, written or jotting method depending on what the problem requires.
- Children estimate and use the inverse operation to check a problem.
- Children solve 2 step problems involving + and -, picking the correct operation and method.
- Children solve simple money and measure problems with fractions and decimals.
- Find 1000 more or 1000 less than a given number.
- Count backwards through zero including negative numbers.
- Recognise the place value of each digit in a 4 digit number.
- Round any number to the nearest 10, 100 or 1000.
- Solve number and practical problems that involve increasingly large positive integers.

Video : <https://www.youtube.com/watch?v=3ihxp2mqnhs>

Year 5: Subtracting with numbers beyond 4 digits including decimals

Children in year 5 will continue to use the compact column method of subtraction to solve problems including those where exchanging is required. They will subtract larger integers and begin to subtract decimal amounts.

$$\begin{array}{r} \overset{2}{\cancel{3}} \overset{10}{\cancel{1}} \overset{0}{\cancel{0}} \overset{6}{\cancel{8}} \overset{6}{\cancel{6}} \\ - \quad \quad 2 \quad 1 \quad 2 \quad 8 \\ \hline 2 \quad 8, \quad 9 \quad 2 \quad 8 \end{array}$$

Children will come across problems where exchanging will need to take place several times to complete the problem.

Once confident with large integers, children will now be ready to move onto decimal numbers including lots in the context of measures and money. Just like addition, it is important that the children line up the decimal point and understand why they are doing this.

$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{10}{\cancel{1}} \overset{6}{\cancel{6}} \overset{8}{\cancel{9}} \cdot \overset{0}{\cancel{0}} \\ - \quad \quad 3 \quad 7 \quad 2 \cdot 5 \\ \hline 6 \quad 7 \quad 9 \quad 6 \cdot 5 \end{array}$$

Add a „zero“ in any empty decimal places to aid understanding of what to subtract in that column.

Key vocabulary:

equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills

- Subtract numbers mentally with increasingly large numbers .
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy .
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Year 6: Subtracting with increasingly large and more complex numbers and decimal values.

In year 6, children need to use mental methods and the compact column method of subtraction to solve an increasingly complex range of calculation including those with integers, those with decimals and those with mixed numbers.

$$\begin{array}{r}
 \cancel{7}^{\circ} \cancel{5}^{\circ} \cancel{10}^{\circ}, 6 \ 9 \ 9 \\
 - \quad 8 \ 9, 9 \ 4 \ 9 \\
 \hline
 6 \ 0, 7 \ 5 \ 0
 \end{array}$$

Children will use the compact method to solve problems involving integers up to 6 digits and beyond and solve problems where they will need to use 'exchanging' several times.

They will also solve problems in context involving increasingly large decimals. They will need to continue using their knowledge of decimal points to line up their numbers and place zeroes in any empty places so they fully understand the value of that.

$$\begin{array}{r}
 \cancel{7}^{\circ} \cancel{10}^{\circ} 5 \cdot \cancel{4}^{\circ} 1 \ 9 \ \text{kg} \\
 - \quad 3 \ 6 \cdot 0 \ 8 \ \text{kg} \\
 \hline
 6 \ 9 \cdot 3 \ 3 \ 9 \ \text{kg}
 \end{array}$$

Key Vocabulary

Equal to, take, take away, less, minus, subtract, leaves, distance between, difference between, how many more, how many fewer/less than, most, least, count back, how many left, how much less is_?, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal place, decimal

Key Skills

- Solve addition and subtraction multi step problems in context, deciding which operations to use and why.
- Read, write, order and compare numbers to at least 10 million and understand the value of each digit.
- Round any whole number up to 10 million to the nearest 10, 100, 1000, 10 000, 100 000, or 1 million.
- Use negative numbers in context and calculate intervals across zero.
- Look at a calculation and decide whether you need to use a mental method, a jotting, a written method or a calculator to solve.

Multiplication

Year 1: Multiply with concrete objects, arrays and pictorial representations.

In year one children will be exposed to many different multiplication based activities in a variety of contexts. Much of this will be repeated addition activities or be linked to counting in 2s, 5s or 10s.

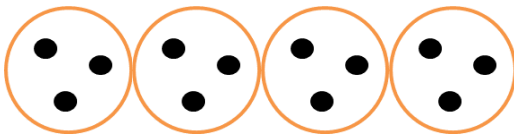
How many legs will 3 teddies have?



$$2 + 2 + 2 = 6$$

There are 3 sweets in one bag.
How many sweets are in 5 bags
altogether?

$$3+3+3+3+3= 15$$



Give children experience of counting equal group of objects in 2s, 5s and 10s.

Present practical problem solving activities involving counting equal sets or groups

Key vocabulary

groups of, lots of, times, array, altogether, multiply, count

Key skills:

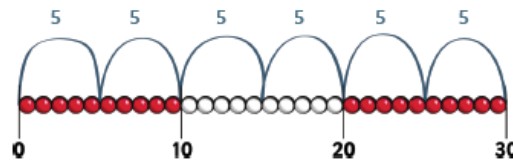
- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

Year 2: Multiplying using arrays and repeated addition- 2,3,4,5,10x table facts

In year 2 children will be aware of simple arrays and pictorial representations and understand what they mean. In year 2 children will develop the knowledge of how to make their own arrays to solve a problem and also how repeated addition on a numberline can get them to a solution.

Counting in steps ('clever' counting)

Count in 2s, 5s, 10s



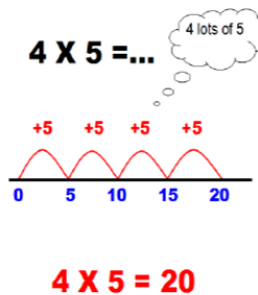
Doubling

Know doubles to double 20

Begin to know doubles of multiples of 5

to 100 E.g. double 35 is 70

Begin to double 2-digits numbers less than 50 with 1s digits of 1, 2, 3, 4, or 5



Use repeated addition on a number line:

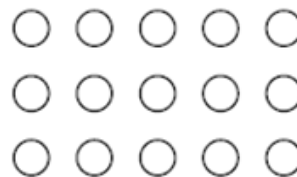
Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.

Using Arrays :

$$5 \times 3 = 3 + 3 + 3 + 3 = 15$$

$$3 \times 5 = 5 + 5 + 5 = 15$$

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as $3 \times \underline{\quad} = 6$.



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times

Key Skills

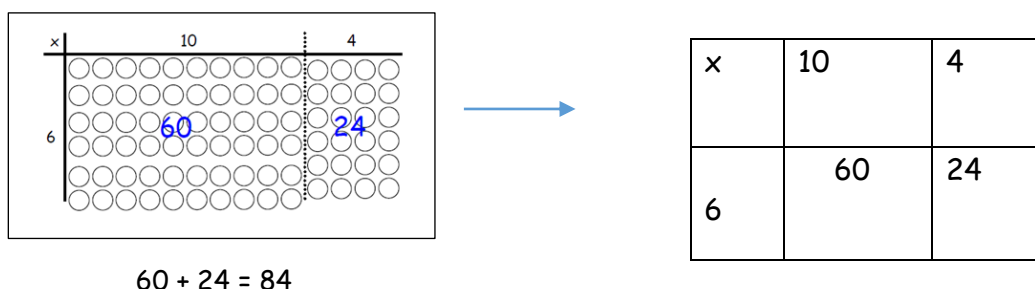
- Count in steps of 2,3 and 5 from zero and in 10s from any number.
- Recall and use multiplication facts for the 2,5 AND 10 times tables.
- Recognise odd and even numbers.
- Write and calculate number statements using the x and = signs.
- Show that multiplication can be done in any order (the commutative law).
- Solve a range of multiplication problems using objects, arrays, repeated addition, mental methods and multiplication facts.
- Use and become familiar with all of the above multiplication language.

Year 3: Multiplying 2 digit numbers by 1 digit numbers

In year 3 children will move on from arrays and start using the grid method of multiplication. It is essential that before children move onto the grid method they are completely confident with all previous methods and have a solid grounding with mental methods and partitioning.

Making the bridge from arrays to grid method

Some children may need to use arrays to help bridge their understanding before going on to grid method



Grid method with greater two digit numbers

Eg. $23 \times 8 = 184$

X	20	3
8	160	24

When children become more confident use grid methods with greater two digit numbers
Use mental skills, if 2×8 is 16..... 20×8 is 160

In year three use of the grid method should reinforce mental knowledge of 3,4 and 8 times table

For more able children teach the column method alongside the grid method when children are confident in the grid method. (see year 4)

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value

Key skills:

- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including 2-digit x single-digit, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using the commutative law (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g using the commutative law ($4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and for missing number

Year 4: Multiplying 2 and 3 digit numbers by a single digit

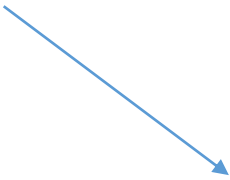
In year 4 children begin by using the grid method confidently to solve problems where a 2 or 3 digit number is multiplied by a one digit number. They need to move on to the use of short multiplication to solve 3 digit number multiplied by 1 digit problems.

Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g:
— 346×9 is approximately $350 \times 10 = 3500$
- Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.
- Recall all times tables up to 12×12

Pupils could be asked to work out a given calculation using the grid, and then compare it to „your“ column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps

x	300	20	7
4	1200	80	28



	3	2	7
x			4
<hr/>			
	1	3	0
		8	
	1	2	

Key vocabulary

groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, inverse

Key skills

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for all multiplication tables up to 12×12 .
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally $3 \times 6 = 6 \times 3$, $2 \times 6 \times 5 = 10 \times 6$, $39 \times 7 = 30 \times 7 + 9 \times 7$.
- Solve problems with increasingly complex multiplication in a range of contexts.

Year 5: Multiplying up to 4 digits by 1 or 2 digits

In year 5 children will continue to use short multiplication to solve increasingly richer problems that involve multiplying by 1 digit. They will then move on to long multiplication for problems that involve multiplying by 2 digits. Approximation will play an important part- with children making approximations before using long multiplication to help check their answer is correct.

$$\begin{array}{r} 3652 \\ \times \quad 8 \\ \hline 29216 \\ 541 \end{array}$$

Children will use short multiplication in a range of increasingly challenging problems. Solving using the grid method and then comparing to the short multiplication method will help cement the children's understanding of the short multiplication method.

	10	8
10	100	80
3	30	24

	1	8	
\times	1	3	
	5	4	
	1	8	0
	2	3	4

When multiplying by more than 1 digit, children need to use long multiplication. Like with short multiplication, they will solve the problem using the grid method first and then make comparisons until their understanding is secure. In the example below the top row shows 18×3 and the bottom shows 18×10 . The final row shows the total of both calculations.

$$\begin{array}{r} 1234 \\ \times \quad 16 \\ \hline 7404 \\ 12340 \\ \hline 19744 \end{array}$$

Once long multiplication methods are secure, children are ready to move on to more challenging problems which require greater levels of mental calculation. The problem to the right show 1234×6 on the top line, 1234×10 on the bottom line and the total of both calculations on the final row.

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square**, **factor**, **integer**, **decimal**, **short/long multi-plication**, **carry**

Key skills

- Identify multiples and factors, using knowledge of **multiplication tables to 12×12** .
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately

Year 6: Consolidating short and long multiplication, multiplying decimals by 1 digit

In year 6 children will consolidate all they know about short and long multiplication before they go to Secondary school. They will also learn the new skill of using short multiplication to multiply decimal numbers to 2 decimal places

	3	.	1	9	
x	8				
<hr/>					
2	5	.	5	2	
	1		7		

When multiplying decimals it is important to remember that the digit you are multiplying by needs to be lined up with the ones digits. As with all decimal work, the decimal points must be lined up and the children need to have a clear understanding why that is.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit**.
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number**.

Key Vocabulary

Groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, multiple, product, tens, units, value, inverse, square, factor, integer, decimal, short/long multiplication, carry, *tenths, hundredths, decimals*

Key Skills

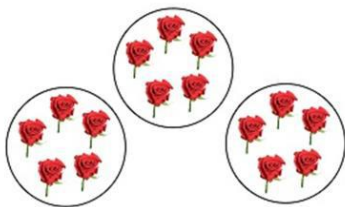
- Multiply up to 4 digits by 2 digits using long multiplication.
- Solve mixed operation and large number problems using mental methods.
- Solve multi step problems involving a range of operations.
- Estimate and approximate answers of problems to improve accuracy.
- Round any integer to the determined level of accuracy.

Division

Year 1: Grouping and sharing small quantities without remainders

As an introduction to division, children in year 1 will solve problems in familiar and relevant contexts where they have to group and share. They will use objects and pictorial representations to solve problems and they will begin to use counting in 2s, 5s and 10s to support their problems solving.

A farmer has 15 roses and shares them between 3 friends. How many roses do they each get?



15 roses shared between 3 = 5 roses each

Bats fly in groups of 2. How many groups of 2 will there be if there are 8 bats?



8 bats shared into groups of 2 = 2 bats in each group

Children need to learn grouping and sharing alongside each other so they understand how they are linked. Grouping will also help children understand how multiplication can be used to solve division problems. Contextual problems will strengthen children's understanding of division.

Key Vocabulary:

share, share equally, one each, two each..., group, groups of, half lots of, array

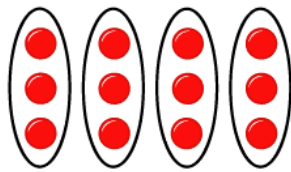
Key skills:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Year 2: Grouping and sharing larger quantities using written methods and symbols

Children will continue to use the methods of sharing and grouping in division with objects to support their understanding of arrays for sharing and grouping and the division numberline for grouping.

Arrays

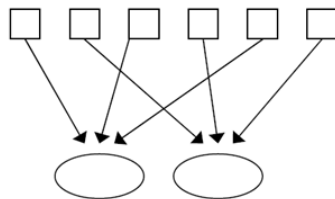


$$12 \div 3 = 4$$

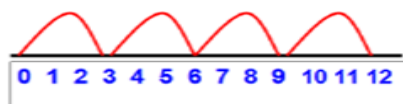
This represents $12 \div 3$, posed as how many groups of 3 are in 12? Pupils should also show that the same array can represent $12 \div 4 = 3$ if grouped horizontally.

Gain a deeper understanding of sharing and grouping

6 sweets shared between 2 people, how many do they each get?



There are 6 sweets, how many people can have 2 sweets each?



$$12 \div 3 = 4$$

Grouping using a number line

Group from zero in equal jumps of the divisor to find out "how many groups of _ in _?". Pupils could use a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?". This is an important method to develop understanding of division as grouping.

Sharing

Grouping

Key Vocabulary

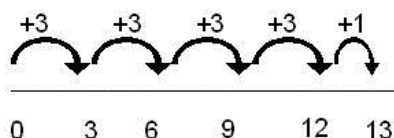
Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

Key Skills

- Count in steps of 2, 3 and 5 from 0.
- Recall and use \times and \div facts for the 2, 5 and 10 times tables.
- Solve division problems and write division number sentences for problems.
- Understand that division is not commutative unlike multiplication.
- Solve increasingly challenging division problems using concrete objects, arrays, and simple written methods such as grouping on a numberline.

Year 3: Dividing 2 digit numbers by 1 digit numbers moving from numberline methods to short division

Children in year 3 will continue to use a numberline to solve division problems and will begin to jump more than one step at a time in the style of 'chunking'. Once confident they will move on to short division without any remainders.



Once confident children will begin to solve problems on a grouping numberline involving bigger numbers. To solve this effectively they will need to subtract chunks of the divisor. As you can see in the image for $92 \div 4$, a step of 10 groups of 4 has been jumped, followed by another step of 10 jumps, finally followed by a step of 3 jumps of 4. This means that in total 4 was jumped 23 times making 23 the answer.

$$\begin{array}{r} 32 \\ 3 \overline{) 96} \end{array}$$

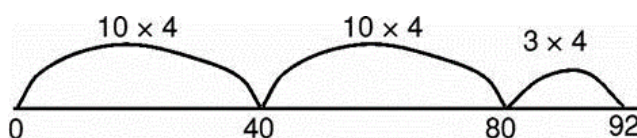
Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple

Key Skills

- Recall and use and \div facts for the 2,3,4,5,6,8 and 10 x tables (using doubling to connect the 2,4 and 8 x tables)
- Solving division problems where a 2 digit number is divided by a 1 digit number using mental and written.
- Solve problems in a variety of contexts including missing number problems.
- Pupils begin to derive related facts e.g. $9 \div 3 = 3$ means $90 \div 3 = 30$ or $90 \div 30 = 3$.
- Pupils develop confidence in written methods, moving from numberlines to short division.

Children will begin to use the grouping numberline method to solve problems with remainders. They will start on zero and write the dividend at the end of their numberline. They will jump in steps of the divisor until they get as close to the end as possible. Whatever is left over is the remainder. Using cubes or arrays alongside the numberline will consolidate understanding.



Once children are confident with numberline methods then they should start work on short division. First of all arrays should be used to show a division calculation, the same calculation should then be shown in the short multiplication method. Place value should be regularly discussed so children realize that they are partitioning the dividend and dividing the units then then tens by the divisor.

Year 4 Consolidating and extending use of short division

Children in year 4 will continue to use short division to solve division problems. They will begin to work on remainders, including problems where there are remainders in the first numbers but not in the final answer.

$$\begin{array}{r} 12 \\ 8 \overline{) 96} \end{array}$$

Once confident with the method of short division, they will move on to problems where the first digit of the dividend is not a multiple of the divisor and therefore a remainder will need to be carried. Children may need to use other equipment to calculate the division and multiplication facts required.

Children who can use short multiplication problems with remainders (but not those in the final answer) are now ready to work on 3 digit problems. Again, there should be remainders in the calculation but never in the final answer.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$$

$$\begin{array}{r} 035 \\ 5 \overline{) 175} \end{array}$$

Once children are confident at dividing with 3 digits, they need to attempt problems where the answer in the first column (hundreds column) is a zero. They may wish to record the hundred initially as this will help them remember its place and the numbers value.

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor

Key Skills

- Recall multiplication and division facts for all numbers to 12×12 .
- Use place value and known facts to derive facts mentally- including multiplying and dividing by 100, 10 and 1.
- Practise mental methods and extend this to three digit numbers using derived facts- e.g. $100 \div 5 = 20$ so $20 \times 5 = 100$.
- Solve two step problems with increasingly harder numbers in a range of contexts, using language to identify the correct operation.
- Correspondence problems should be introduced such as 3 cakes are shared equally between 10 children, 1 man has 6 cats so how many cats do 3 men have etc.

Year 5 Extending use of short multiplication to 4 digits and remainders

Children in year 5 will use short division to solve problems up to 4 digits long. For the first time they will use short division to solve problems that have a remainder in the final answer.

In year 5 children will begin to solve division problems where a number up to 4 digits is divided by a single digit number including answers with remainders. These division problems need to be contextual so the children learn how to express the remainder- as a number, a fraction, a decimals, rounded up or rounded down.

$$\begin{array}{r} 0663r5 \\ 8 \overline{) 535029} \end{array}$$

If some children are confident they can move onto long division (see year 6)

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime)

Key Skills

- Multiply and divide numbers mentally, using known facts.
- Identify multiples and factors, including all factor pairs of a number and common factors between 2 numbers.
- Solve \times and \div problems where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and decimals by 10, 100 and 1000.
- Use vocabulary of prime numbers, prime factors and composite numbers.
- Work out whether a number up to 100 is prime and know all prime numbers to 30.
- Use and understand multiplication and division as inverses.
- Present division with remainders answers differently, showing the remainder as a fraction, decimal or whole number by rounding.
- Solve problems with a combination of all four operations including fraction scaling problems and problems involving simple rates.

Year 6 Using short division to divide 4 digit numbers and express remainders as decimals and long division for dividing 2 digit numbers

In year 6, children will use short division to divide decimal numbers by single digit numbers. The final step of division will be long division which will be used to divide numbers by 2 digits.

$$\begin{array}{r} 0812.125 \\ 8 \overline{)6497.000} \end{array}$$

The focus in year 6 is not so much the method of short division but how the remainders are expressed- children need to express remainders as decimals and fractions- depending on the context of the question.

To divide by 2 digit numbers, the children will use the method of long division. The example to the right clearly shows the method in the 'Burger' steps, where as the example to the left shows what a completed method would look like. Any remainders would need to be expressed in a way that matched the context of the problem.

$$\begin{array}{r} 291 \\ 45 \overline{)13095} \\ \underline{90} \\ 409 \\ \underline{405} \\ 45 \\ \underline{45} \\ 0 \end{array}$$

Divide:	
Multiply:	
Subtract:	
Bring Down:	
Repeat:	

Key Vocabulary

Share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, quotient, prime number, prime factors, composite number (non-prime), common factor

Key Skills

- Use multiplication and division facts up to 12 x 12 to solve more complex problems.
- Decide when to use short or long division and interpret remainders in a way that is appropriate to the problem.
- Perform mental calculations for problems involving large numbers and mixed calculations.
- Identify common factors, common multiples and prime numbers.
- Use estimation to check answers to calculations and determine accuracy.
- Use written methods of division to solve decimal problems up to 2 decimal places.
- Solve problems which require rounding to 10. 100. 1000 and beyond.